SOME PROBLEMS OF THE HYDRODYNAMICS OF A HORIZONTAL CYLINDER WITH NATURAL CONVECTION

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The article examines the special features of vortex flow in the vicinity of the upper critical point, and approximating dependences are obtained for the point of detachment from the data of numerical solution and experiments.

The curvature of a horizontal cylinder determines a number of hydrodynamic and heat exchange features when natural convection is involved. This manifests itself first of all in the detachment of the boundary layer and the formation of vortex flow in the rear region. Analysis of the hydrodynamics and heat exchange from the positions of the boundary layer theory has limited application and is correct only up to the point of detachment. Therefore, to obtain an exhaustive picture of the development of the flow and of the changes of the local heat transfer coefficient over the entire circumference of the cylinder, including the eddy region, we have to use the full system of Navier–Stokes and energy equations. Moreover, such an approach enables us to determine the position of the point of detachment, and consequently also the limits of applicability of the solutions and calculation formulas obtained in the approximations of the boundary layer.

Numerical calculation of the full bivariate Navier–Stokes and energy equations was carried out in the range of Grashof numbers $10^2 \leqslant Gr_d \leqslant 10^7$ and Prandtl numbers $0.01 \leqslant Pr \leqslant 100$ with the boundary conditions on the wall T_W = const and q_W = const. As a result of the calculation we obtain the fields of the stream function of $\bar{\psi}$, of the temperature $\bar{T}$, and of the speed $\bar{V}$ near the cylinder. By the isolines of the stream function and the speed profiles we can analyze the hydrodynamic flow pattern and the change in the type of flow in different regimes.

We will examine the change of the hydrodynamic flow pattern of air (Pr = 0.72) with different Gr_d numbers (Fig. 1). In the vicinity of the upper critical point we find detachment of the flow in the form of a stable vortex that is relatively small in the radial direction ($\sim$0.1 R) and extensive circumferentially. With small Grashof numers ($\sim 10^3$-10^4) the vortex motion is more developed (Fig. 1a, b); with increasing intensity of convection we find a tendency to its disappearance. With $Gr_d \sim 10^7$, the dimensions of the vortex were so small that we did not manage to determine them with the difference grid (21 × 111) that we used. Similar hydrodynamic flow patterns were observed visually by Hyman et al. [1].

The temperature fields (Fig. 1) in a liquid fully correspond to the hydrodynamic patterns. The different density of isotherms over the perimeter of the cylinder indicates that the local heat transfer coefficient has its maximum value in the front part of the cylinder; in the rear part the isotherms are extended, and to this corresponds the minimum value of the heat transfer coefficient.

From the results of the numerical calculation we established the position of the point of detachment of the boundary layer; from this point on there appears reverse flow. As the point of detachment we took the point in which $\partial \bar{V}_\varphi / \partial r = 0$.

Figure 2a shows the change of the position of the point of detachment in dependence on the number Gr_d calculated from the cylinder diameter, with Pr = 0.72 and T_W = const. The same figure also shows the experimental values of the position of the point of detachment, obtained with the aid of a laser anemometer made by DISA.

The speed was measured in differential Doppler regime with direct scatter. The measuring volume forming at the point of intersection of the laser beams is an ellipsoid of revolu-

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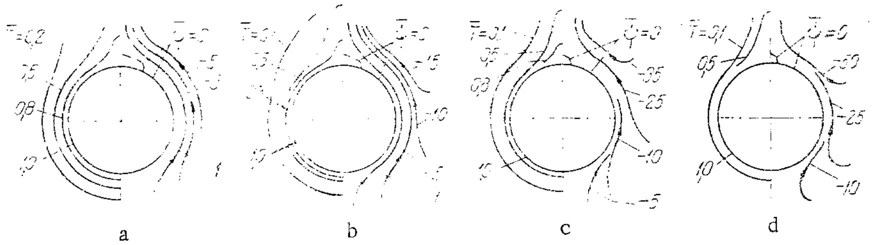

Fig. 1. Fields of the stream [$\bar{\Psi}$] and of the temperature function [$\bar{T}$] with different flow regimes: a) $Gr_d = 3.2 \cdot 10^3$; b) $3.2 \cdot 10^4$; c) $3.2 \cdot 10^5$; d) $3.2 \cdot 10^6$.

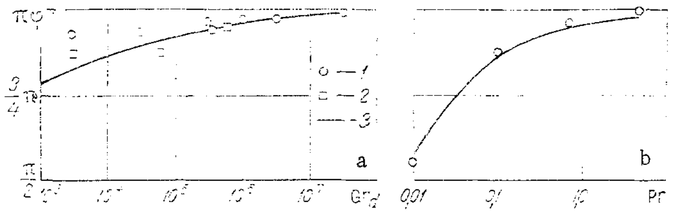

Fig. 2. Position of the point of detachment in dependence on the numbers Gr_d (a) and Pr (b) with T_w = const: 1) results of numerical solution; 2) experiment; 3) formula (1).

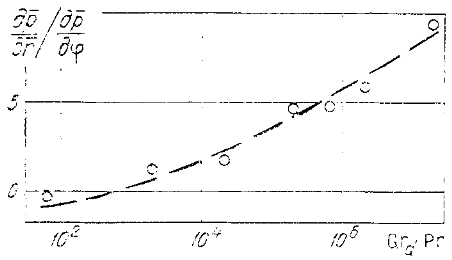

Fig. 3. Ratio of the components of the pressure gradient in the vicinity of the upper critical point (rear part, $\varphi = 175°$) with Pr = 0.72.

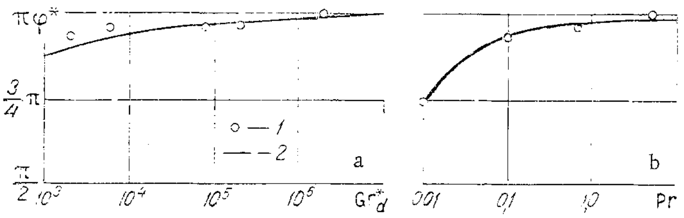

Fig. 4. Position of the point of detachment with q_w = const: 1) numerical solution; 2) formula (4).

tion with the diameters 0.225 and 1.8 mm. The light-scattering particles were tobacco smoke, which was uniformly distributed throughout the chamber. The size of the particles was 0.4-1.0 μm. To determine the direction of the speed, we used the Doppler frequency shift with the aid of a speed-direction adapter. During the speed measurements, the laser beams intersected in a plane parallel to the generatrix of the cylinder. For that the measuring volume was set on the cylinder surface by rotating the beam splitter around its axis and by a coordinate table on which the optical system of the instrument was mounted. Scanning of the speed profile was effected with an accuracy of 0.025 mm along any of three coordinates. Possible errors in determining the coordinates of the measuring volume were evaluated by calculation; they did not exceed 3% of the minor diameter of the measuring volume. Such an error may arise when there are considerable temperature gradients near the cylinder wall (of the order of magnitude of 500°C per 1 cm). In our case the maximum temperature gradient did not exceed 30°C.

The position of the point of detachment of the boundary layer was found from speed profiles measured with T_w = const. First the position of the zone of detachment was determined. The speed profiles in doing that were measured in steps $\Delta\varphi = 5°$. Then the position of the point of detachment was found more accurately by successive measurements in steps of $\Delta\varphi = 2°$. The angular accuracy of the measurements did not exceed ±2°. More accurate measurements cannot be ensured because the place of detachment shifts continuously in consequence of the instability of the flow in the region of detachment. Figure 2b shows the position of the point of detachment in dependence on the change of the Pr number with $Gr_d = 10^6$.

We managed to approximate the numerical calculations and the experimental data by formula (1), which made it possible to calculate, with some approximation, the position of the point of detachment φ^* with T_w = const:

$$\frac{\varphi^*}{\pi} = \left(\frac{Gr_d^{1/3}}{6.4 + Gr_d^{1/3}}\right)^{1/2} \left(\frac{Pr}{0.04 + Pr}\right)^{1/3}, \tag{1}$$

where the angle φ^* is read off the front point of the cylinder. Formula (1) is correct for numbers $Gr_d > 10^3$.

The peculiarities of the flow in the front region are chiefly due to the change of the components of the excess pressure gradient $\frac{\partial \bar{p}}{\partial \varphi}$ and $\frac{\partial \bar{p}}{\partial \bar{r}}$. To determine them, it is indispensable to solve the inverse problem — to find the distribution $\frac{\partial \bar{p}}{\partial \varphi}$ and $\frac{\partial \bar{p}}{\partial \bar{r}}$ on the cylinder surface from the known speed and temperature fields. Since on the cylinder surface the radial ($\bar{V}_r$) and tangential ($\bar{V}_\varphi$) components of the speed are equal to zero, the relations for determining the pressure gradients have the form:

$$\left.\frac{\partial \bar{p}}{\partial \bar{r}}\right|_{\bar{r}=1} = -\frac{Gr_d Pr^2}{8}\,\bar{T}\cos\varphi + Pr\left(\frac{\partial^2 \bar{V}_r}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\,\frac{\partial \bar{V}_r}{\partial \bar{r}}\right)\Bigg|_{\bar{r}=1}, \tag{2}$$

$$\left.\frac{\partial \bar{p}}{\partial \varphi}\right|_{\bar{r}=1} = \bar{r}\left[\frac{Gr_d Pr^2}{8}\,\bar{T}\sin\varphi + Pr\left(\frac{\partial^2 \bar{V}_\varphi}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\,\frac{\partial \bar{V}_\varphi}{\partial \bar{r}}\right)\right]_{\bar{r}=1}. \tag{3}$$

Numerical calculations of Eqs. (2) and (3) show that the pressure gradients $\frac{\partial \bar{p}}{\partial \varphi}$ and $\frac{\partial \bar{p}}{\partial \bar{r}}$ are of commensurable magnitudes. Consequently, Hermann [2] was not entirely right when he assumed that with free convection near a horizontal cylinder $\frac{\partial \bar{p}}{\partial \bar{r}} = 0$ and that the equation of motion in the projection on the radial direction may be excluded from the statement of the problem.

If we examine the relations $\frac{\partial \bar{p}}{\partial \bar{r}}$ and $\frac{\partial \bar{p}}{\partial \varphi}$ in the rear region, we find an explanation for the appearance and disappearance of the vortex motion. For instance, with $Gr_d = 10^6$ in the rear region, $\frac{\partial \bar{p}}{\partial \bar{r}} > 0$. That means that at some distance from the cylinder surface the pressure of the liquid is greater than at the surface. Thus, this difference in pressures exerts a stabilizing effect on the flow. Conversely, a positive pressure gradient $\frac{\partial \bar{p}}{\partial \varphi}$, causes the moving layer of liquid to be forced away from the cylinder surface. With small numbers Gr_d the ratio $\frac{\partial \bar{p}}{\partial \bar{r}} \Big/ \frac{\partial \bar{p}}{\partial \varphi}$ in the rear part (Fig. 3) is smaller than unity, and with $Gr_d = 10^2$ it becomes negative; the stabilizing effect of the gradient $\frac{\partial \bar{p}}{\partial \bar{r}}$ manifests itself weakly, whereas a positive pressure gradient $\frac{\partial \bar{p}}{\partial \varphi}$ causes the moving layer of liquid to be forced away from the cylinder surface. With small numbers Gr_d this leads to the appearance of relatively large vortices in the rear part. With increasing Gr_d (intensity of heat exchange) the stabilizing effect $\frac{\partial \bar{p}}{\partial \bar{r}}$ increases, and as a result the size of the zone with vortex flow decreases, and then it vanishes altogether.

Similar calculations were also carried out for the case of q_w = const. When the Grashof numbers $Gr_d^* < 10^5$ there is vortex flow in the rear part. When $Gr_d^* > 10^5$, the dimensions of the region with vortex flow become so small that even with $Gr_d^* = 10^6$ the selected finite-difference grid does not make it possible to register them. The reduced size of the zone with vortex motion of the liquid in the rear part is also due to the stabilizing effect of the radial pressure gradient on the flow.

Hydrodynamic calculations for q_w = const revealed the effect of the numbers Gr_d^* and Pr on the position of the point of detachment (Fig. 4). We want to point out that as compared with T_w = const, the position of the point of detachment with constant heat flow on the wall shifts upstream, i.e., the flow with q_w = const is stabler.

The data of numerical calculation of the position of the point of detachment with q_w = const were approximated by the formula

$$\frac{\varphi^*}{\pi}=\left(\frac{Gr_d^{*1/3}}{3+Gr_d^{*1/3}}\right)^{1/2}\left(\frac{Pr}{0.012+Pr}\right)^{1/3}, \tag{4}$$

which is suitable for numbers $Gr_d^* > 10^3$.

NOTATION

Gr_d is the Grashof number with characteristic dimension d (cylinder diameter); Pr, Prandtl number; T_w, specified temperature of the cylinder surface; q_w, heat flow on the cylinder surface; $\bar{\psi} = \psi/a$, dimensionless function of the flow; a, thermal diffusivity of the liquid; $\bar{V} = VR/a$, dimensionless speed of the liquid; $\bar{p}$, dimensionless static excess pressure; $\bar{r} = r/R$, φ, dimensionless radius and angle, respectively, in the polar coordinate system; φ^*, angle determining the position of the point of detachment; Gr_d^*, modified Grashof number in which the complex $q_w d/\lambda$ is used instead of the temperature difference ΔT; and λ, thermal conductivity of the liquid. Subscripts: d, characteristic dimension (diameter); w, wall; and r, φ, projections of the speed on the radial and tangential direction, respectively.

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